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XLIV. Fig. 26.
$AFHd = OABc$ is equivalent to $AEOcC$.
$BeH = KBT$ is equivalent to $KBCb + ODc$. $Bed = KLb$.
$\therefore$ $ABHF$ is equivalent to $ACDE + BKLC$.

XLV. Fig. 26.
$HaW = KLb$. $AFHaWB$ is equivalent to $ACBWF$ is equivalent to $ONPfA$ is equivalent to $ONCA + NPfC$ is equivalent to $ACDE + BKbC$.
$\therefore$ $ABHF$ is equivalent to $ACDE + BKLC$.

[To be Continued.]

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D. (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from April Number.]

PROPOSITION XXVII. *If a straight AX (Fig. 32.) drawn at any however small angle from the point A of AB, must at length meet (anyhow at an infinite distance) any perpendicular BX, which is supposed erected at any distance from this point A upon the secant AB: I say there will then be no more place for the hypothesis of acute angle.*

PROOF. From any point K chosen at will in AB near the point A, the perpendicular KL is erected to AB, which certainly (from Cor. II. of the preceding proposition) meets AX at a finite or terminated distance in some point L. But now it holds that there may be assumed in KB portions KK each equal to a certain assignable length R, and these more than any assignable finite number; since indeed the point B can be situated, in accordance with the present supposition, at however great a distance from this point A.

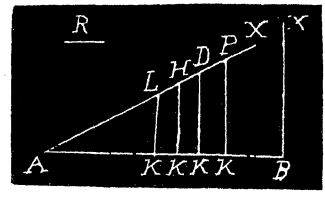

Fig. 32.

And accordingly from the other points K are erected to AB perpendiculars KH, KD, KP, which all (from the aforesaid corollary) meet the straight AX in certain points H, D, P; and so about the remaining points K uniformly designated toward the point B.

It holds secondly (from Eu. I. 16) that the angles at the points L, H, D, P will all be obtuse toward the parts of the points X; and just so (from Eu. I. 13) the angles at the aforesaid points will all be acute toward the point A.

Therefore (from Cor. II. after 3 of this) the side KH will be greater than the side KL ; the side KD greater than the side KH ; and so always proceeding towards the points X.

It holds thirdly that the four angles together of the quadrilateral $KLHK$ will be greater than the four angles together of the quadrilateral $KHDK$: for this in like case has already been demonstrated in XXIV of this.

It holds fourthly that the same is valid likewise of the quadrilateral $KHDK$ in relation to the quadrilateral $KDPK$; and so on always, proceeding to quadrilaterals more remote from this point A.

Since therefore are present (as in XXV of this) as many quadrilaterals described in the aforesaid mode, as there are, except the first LK, perpendiculars let fall from points of AX to the straight AB, it will hold uniformly (if we assume nine perpendiculars of this sort let fall, besides the first) the sum of all the angles which are comprehended by these nine quadrilaterals will exceed 35 right angles ; and therefore the four angles together of the first quadrilateral $KLHK$, which indeed in this regard has been shown the greatest of all, will fall short of four right angles by less than the ninth part of one right angle. Wherefore, these quadrilaterals being multiplied beyond any assignable finite number, proceeding always toward the parts of the points X, it holds in the same way (as in the same already recited theorem) that the four angles together of this stable quadrilateral $KHLK$ will fall short of four right angles less than any assignable little portion of one right angle.

Therefore these four angles together will be either equal to four right angles, or greater.

But then (from XVI of this) is established the hypothesis either of right angle or of obtuse angle ; and therefore (from V and VI of this) is destroyed the hypothesis of acute angle.

So then it holds, that there will be no place for the hypothesis of acute angle, if the straight AX drawn under however small angle from the point A of AB must at length meet (anyhow at an infinite distance) any perpendicular BX, which is supposed erected at any distance from this point A upon this secant AB.

Quod erat etc.

SOME DIVISIBILITY TESTS.

By WM. E. HEAL, Member of the London Mathematical Society, Marion, Indiana.

In the *Educational Times* for March, 1897, Professor Sylvester proposed the following problem : "If the digits r in number of any integer N read from left to right be multiplied repeatedly by the first r terms of the recurring series